Quiz 4

(1) Let $f(x) = 20 + x - x^2$ and $g(x) = x^2 - 5x$. Set up an integral that gives the area between these graphs for $x \in [4, 8]$.

\[ f(x) = 20 + x - x^2 \]
\[ g(x) = x^2 - 5x \]
\[ A = \int_4^8 [f(x) - g(x)] \, dx + \int_5^8 [g(x) - f(x)] \, dx \]

(2) For $x \in [1, 4]$, consider the solid whose base is the region bounded above by $f(x) = x^3$ and below by the $x-$axis. Suppose that the cross sections perpendicular to the $x-$axis are semi-circles. Set up an integral to find the volume of this solid.

\[ f(x) = x^3 \]
\[ A(x) = \frac{\pi}{8} \int_1^4 \, dx \]

(3) Consider the same problem as the last one, only now suppose that the cross sections perpendicular to the $x-$axis are equilateral triangles. Again, set up an integral to find the volume of the solid.

\[ A = \frac{1}{2} \cdot b \cdot h \]
\[ b = x^3 \]
\[ h = \frac{1}{3} \cdot x^3 \]
\[ V = \int_1^4 \frac{1}{2} (x^3) \left( \frac{1}{3} \cdot x^3 \right) \, dx \]
(4) Consider the region bound above by the graph of \( x^2 + x \) and below by the \( x \)-axis, where \( x \) is in \([1, 3]\). Suppose you revolve this region around the \( x \)-axis. Set up an integral to find the volume of the resulting solid.

\[
\text{Disk Method: } A(x) = \pi r^2 = \pi (x^2 + x)^2
\]

\[
V = \int_{1}^{3} A(x) \, dx = \int_{1}^{3} \pi (x^2 + x)^2 \, dx
\]

(5) Suppose you repeat the last problem, only this time you revolve around the line \( y = 12 \). Set up an integral to find the volume of the resulting solid.

\[
\text{Washer Method}
\]

\[
A(x) = \pi \left[ R^2 - r^2 \right] = \pi \left[ 12^2 - \left(12 - (x^2 + x) \right)^2 \right]
\]

\[
V = \int_{1}^{3} A(x) \, dx = \pi \int_{1}^{3} \left[ 12^2 - \left(12 - (x^2 + x) \right)^2 \right] \, dx
\]