Quiz 2

(1) Calculate the area between the $x$–axis and the graph of $f(x) = x^3$ over the interval $[2, 4]$.

\[
\int_2^4 x^3 \, dx = \frac{4^4}{4} - \frac{2^4}{4} = 4 - 2 = 2
\]

(2) Calculate $\int_e^{-1} \frac{1}{x} \, dx = (\ln|x|) \bigg|_e^{-1} = \ln(1) - \ln(e) = 0 - 1 = -1$

(3) Calculate $G(16)$, where $dG/dt = t^{-1/2}$ and $G(9) = -5$.

\[
G = 2t^{1/2} + C
\]

\[
G(9) = 2 \cdot 9^{1/2} + C = 6 + C = -5 \implies C = -11
\]

\[
G(16) = 2 \cdot 16^{1/2} - 11 = 8 - 11 = -3
\]

(4) Recall that in the proof of the FTC, we wrote

\[
F(b) - F(a) = (F(x_1) - F(a)) + \cdots + (F(b) - F(a)).
\]

Is this formula valid for any $F$, or just continuous $F$?

For some $F$. (Since for any $F$, the total vertical displacement between $a$ and $b$ is the sum of the vertical displacements along the partition.)