Problem 1  Suppose you flip a coin twice. What is the conditional probability that both flips land on heads, given that at least one does?

\[ P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{1 \cdot \frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \]

Problem 2  Suppose you are playing bridge with a regular deck that has been augmented by the addition of 48 blank cards. (Never mind the precise rules of this game for the moment.) There are thus a total of 13 of each suite, embedded in a much larger ambient deck. Suppose that you know that the North-South partners have precisely 5 hearts. What is the conditional probability that the Western player has precisely 4 hearts?

\[ \frac{100 \text{ cards total: } \Rightarrow 25 \text{ for queen}}{\binom{50}{5} \text{ ways of choosing 5 hearts}} \]

\[ \Rightarrow \frac{\binom{8}{1} \cdot \binom{42}{4}}{\binom{52}{5}} \text{ ways of choosing remaining cards} \]

Problem 3  Suppose you roll two dice, and know that the sum is 7. What is the conditional probability that you rolled a 4?

\[ P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{\frac{1}{11} \cdot \frac{1}{36}}{\frac{6}{36}} = \frac{1}{6} \]

Problem 4  An urn contains 10 balls, of which 6 are red. Suppose you draw 4 balls (with replacement.) What is the conditional probability that the first and third balls were red, given that your sample draw contains exactly three red balls?

\[ P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{\frac{12}{25} \cdot \frac{9}{25}}{\frac{27}{25}} = \frac{3 \cdot 12}{3 \cdot 27} = \frac{1}{3} \]