Quiz 1

**Problem 1** In order to study effectively, a class of 15 students divides itself into three study groups of five students each. How many different divisions are possible?

There are \( \frac{15!}{5! \cdot 5!} \) ways of choosing the groups if the order of the groups matters. Since it does not, there are \( \frac{(5!)^3 \cdot 3!}{5!} \) divisions.

**Problem 2** State the binomial theorem.

\[
(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}
\]

**Problem 3** Use the binomial theorem to prove that \( \sum_{k=0}^{n} \binom{n}{k} = 2^n \).

Setting \( x = y = 1 \) in the binomial theorem yields

\[
2^n = (1+1)^n = \sum_{i=0}^{n} \binom{n}{i} = \sum_{i=0}^{n} \binom{n}{i}.
\]

**Problem 4** Use the previous result to show that if a set has \( n \) objects, then its powerset (i.e. the set of all its subsets) has size \( 2^n \). (Hint: how many subsets of size \( k \) are there?)

There are \( \binom{n}{k} \) ways of choosing a subset of size \( k \). Since the total \( 2^n \) subsets is the sum of the \( k \) of subsets of each size, there are \( \sum_{k=0}^{n} \binom{n}{k} = 2^n \) subsets all together.

**Problem 5** How many different letter arrangements can be formed from the letters PIGDOG?

There are 6! ways of ordering the letters, but each ordering involves 2 G’s which can be permuted without changing the word. There are thus \( \frac{6!}{2!} = \binom{6}{2} \) words.