Chapter 5: Continuous Random Variables
Density Functions

• The continuous-RV analogue of a probability mass function is the *density function* \( f(x) \), defined via

\[
P\{X \in B\} = \int_B f(x) \, dx
\]
Examples of Density Functions

• Uniform:

\[ f(x) = \begin{cases} 
  \frac{1}{b - a} & \text{if } a \leq x \leq b \\
  0 & \text{elsewhere}
\end{cases} \]

**Example 1:** Let \( X \) be distributed uniformly between 0 and 1. What is the probability that \( X \) lies in the interval \([1/4, 5/8]\)?
Examples of Density Fncts (Cont)

• Normal

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

• Features:
  
  – Not analytically tractable (i.e. need a computer or a table to work out probabilities)
  
  – Symmetric
  
  – Two parameter family
Examples of Density Functions (Cont)

• Example 1: Let $X$ be distributed normally, $N(0,1)$. What is the probability that $X$ is positive?

• Example 2: Let $X$ be distributed normally $N(0,1)$. How is $Y = aX + b$ distributed?

• Example 3: What is the probability that a random variable distributed $N(3,3)$ is between 0 and 6?
Examples of Density Functions (Cont)

• Exponential:

\[ f(x) = \begin{cases} 
\lambda e^{-\lambda x} & x \geq 0 \\
0 & \text{elsewhere} 
\end{cases} \]

(Note: often the model used for the amount of time until an event occurs, i.e. an earthquake, a war, etc.)
Examples of Density Functions (Cont)

• Exponential (cont)

• Example 1: Suppose the length of a phone call is exponential with parameter $\lambda = 0.1$. If someone arrives immediately ahead of you at a phone booth, what is the probability that you’ll have to wait more than 10 minutes?
Expectation (Mean) and Variance of a Continuous RV

• The **expectation** of a continuous random variable is given by

\[ E[X] = \int_{-\infty}^{\infty} x f(x) \, dx \]

• The **variance** of a continuous random variable is given by

\[ Var[X] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx \]
Examples of Mean and Variance

• Calculate the mean and variance of a uniform random variable on \([0,1]\).

• Calculate the mean and variance of a normal RV with parameters 0 and 1.

• Calculate the mean and variance of an exponential RV with parameter \(\lambda\).