Chapter 6 Practice Problems

September 22

(1) Two fair dice are rolled. Find the joint probability mass function of $X$ and $Y$ when
   (a) $X$ is the largest value obtained on any die and $Y$ is the sum of the values
   (b) $X$ is the value on the first die and $Y$ is the larger of the two values.

(2) The joint probability density function of $X$ and $Y$ is given by
    \[ f(x, y) = ce^{-y(x^2 - y^2)}, \quad -y \leq x \leq y, 0 < y < \infty \]
   (a) Find $c$
   (b) Find the marginal densities of $X$ and $Y$
   (c) Find $E(x)$

(3) The number of people that enter a drugstore in a given hour is Poisson with parameter $\lambda = 10$. Compute the conditional probability that at most 3 men entered the drugstore, given that 10 women entered in that hour. What assumptions have you made?

(4) The joint density of $X$ and $Y$ is given by
    \[ f(x, y) = \begin{cases} 
      xe^{-(x+y)} & x > 0, y > 0 \\
      0 & \text{otherwise}
    \end{cases} \]

   Are $X$ and $Y$ independent?

(5) Jill’s bowling scores are approximately normally distributed with mean 170 and standard deviation 20, while Jack’s scores are approximately normally distributed with mean 160 and standard deviation 15. If Jack and Jill each bowl one game, then assuming their scores are independent random variables, approximate the probability that
   (a) Jack’s score is higher
   (b) the total of their scores is above 350.

(6) The joint probability mass function of $X$ and $Y$ is given by
    \[ p(1,1) = \frac{1}{8} p(1,2) = \frac{1}{4} \]
    \[ p(2,1) = \frac{1}{8} p(1,2) = \frac{1}{2} \]
   (a) Compute the conditional mass function of $X$ given $Y = i$, $i = 1, 2$.
   (b) Are $X$ and $Y$ independent?

(7) If $X$ and $Y$ are independent exponential random variables, each having parameter $\lambda$, find the joint density function of $Z_1 = X + Y$ and $Z_2 = e^X$.