Section 3.5

Chain Rule and Parametric Functions
Chain Rule

\((f \circ g)'(x) = f'(g(x)) \cdot g'(x)\)

Alternative notation:

\[ y = f(u), \ u = g(x) : \quad \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \]
Examples

• Find $\frac{d}{dx} \sin(x^3 + e^x)$
Examples

• Find \( \frac{d}{dx} \sin(x^3 + e^x) \)

• Find \( \frac{d}{dx} e^{\cos 3x} \)
Examples

• Find \( \frac{d}{dx} \sin(x^3 + e^x) \)

• Find \( \frac{d}{dx} e^{\cos 3x} \)

Find \( \frac{d}{dx} (\tan(2 + 3 \cos x)) \)
Examples

Find \( \frac{d}{dx} (x^3 + 3x^5)^8 \)

Find \( \frac{d}{dx} \frac{1}{\sqrt{x^2 + 7}} \)

Find \( \frac{d}{dx} e^{\ln x} \) (Note: this can be used to show that the derivative of \( \log(x) \) is \( 1/x \).)
Parametric Curves

• Definition: a parametric curve is a set of points \((x(t), y(t))\), where \(x(t)\) and \(y(t)\) are functions of an independent variable \(t\).

• Example: \(x = t, \ y = t^2\)
Slopes of Parametric Curves

• The slope of the tangent line to a parametric curve is given by

\[ \frac{dy}{dx} = \frac{dy/\ dt}{dx/\ dt} \]

• The second derivate is given by:

\[ \frac{d^2y}{dx^2} = \frac{dy'/\ dt}{dx/\ dt} \]
Examples

• \( x(t) = \cos(t) \), \( y(t) = \sin(t) \). Find \( \frac{dy}{dx} \) at the point \( P = (0,1) \).

**Solution**: The point \( P = (0,1) \) is achieve for a value of \( t = \frac{\pi}{2} \). Evaluating the derivatives

\[
\frac{dy}{dt} = -\sin t \quad \frac{dx}{dt} = \cos t
\]

at \( \frac{\pi}{2} \) and forming the quotient yields \( \frac{dy}{dx} = 0 \).