Problem 1  State the Mean Value Theorem

Let \( f \) be continuous on \([a, b]\), differentiable on \((a, b)\). Then

\[ \exists c \in [a, b] \text{ such that} \]

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

Problem 2  Does the function \( f(x) = x^{1/3} \) satisfy the hypotheses of the mean value theorem on the interval \([0, 1]\)? Why or why not?

Yes. \( f(x) \) is continuous on \([0, 1]\)

\[ f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}} \text{ is differentiable on } (0, 1) \]

Problem 3  Find the intervals on which \( f(x) = -x^3 + 2x^2 \) is increasing or decreasing.

\[ f'(x) = -3x^2 + 4x \]

\[ x \begin{array}{ll} - \infty & x \in (0, \frac{4}{3}) \\ 0 & \frac{4}{3} < x \end{array} \]

So \( f \) is increasing on \((0, \frac{4}{3})\), decreasing on \((-\infty, 0) \cup (\frac{4}{3}, \infty)\).

Problem 4  Find the absolute maximum and minimum values of \( f(x) = e^x - 2x \) on \([0, 1]\).

\[ f'(x) = e^x - 2 \Rightarrow e^x = 2 \Rightarrow x = \ln 2 \]

\[ f''(x) = e^x > 0 \text{ everywhere} \Rightarrow \ln 2 \text{ is a min} \]

\[ f(0) = e^0 - 0 = 1 \quad \Delta(1) = e^1 - 2 = 0.7 \Rightarrow x = 1 \text{ is a max} \]

Problem 5  Draw a picture of monkey in a sailor suite. Find the critical points.