Problem 1  Suppose that the edge lengths $x$, $y$, and $z$ of a rectangular box are changing at the following rates:
\[
\frac{dx}{dt} = 1 \text{ m/s} \quad \frac{dy}{dt} = -2 \text{ m/s} \quad \frac{dz}{dt} = 1 \text{ m/sec}
\]
a) Find the rate at which the volume is changing at the instant when $x = 4$, $y = 3$, and $z = 2$.

\[
V = x \cdot y \cdot z
\]
\[
\frac{dV}{dt} = \frac{d}{dt}(x \cdot y \cdot z) = x'y \cdot y \cdot z + x \cdot y' \cdot y \cdot z + x \cdot y \cdot z' = y'y + x'y + z'
\]
\[
\left. \frac{dV}{dt} \right|_{(x=4, y=3, z=2)} = 6 - 16 + 12 = 2
\]

b) Find the rate at which the diagonal $s = \sqrt{x^2 + y^2 + z^2}$ is changing at the same instant.

\[
\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}} \cdot \left[ 2x \cdot x' + 2y \cdot y' + 2z \cdot z' \right]
\]
\[
= \frac{1}{2\sqrt{16 + 9 + 1}} \cdot [2 \cdot 4 \cdot 1 + 2 \cdot 3 \cdot (-2) + 2 \cdot 2 \cdot 2]
\]
\[
= \frac{1}{8} \cdot [8 - 12 + 4] = 0
\]

Problem 2  Circle any of the following which are necessarily true:

a) If $f$ is continuous on an interval $(a, b)$, then $f$ attains both an absolute maximum and an absolute minimum in $(a, b)$.

b) If a function $f$ attains both an absolute maximum and an absolute minimum in $[a, b]$, then $f$ is continuous on $[a, b]$.

c) If a function $f$ has a local maximum or minimum at a point $x = c$, then $f'(c) = 0$.

d) The extreme values of a function differentiable on the interval $[a, b]$ will be attained either at critical points or at the endpoints.

Problem 3  Find where the extreme values of the function $y = x^3 + x^2 - 8x + 5$ occur.

\[
y' = 3x^2 + 2x - 8 = 0
\]
\[
\Rightarrow \quad \frac{2}{3 \sqrt{2}} \cdot x = -2 \cdot 3 \sqrt{4 - 4 \cdot 3 \cdot (-8)} = -2 \cdot 3 \cdot \frac{100}{2.3}
\]
\[
= \frac{-2 \div (1.0)}{2}
\]